Investigating the efficiency of ore-waste classification by linear and nonlinear kriging

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Outline

- Introduction
- Methodology
- Presentation of case study
- Discussion and conclusions

Introduction

- Linear geostatistics
- Limits of linear geostatistics
- Non-linear geostatistics
- Ore/waste classification

Objective

The objective of this work is understanding how would be efficinet the classification of reservoir into ore/waste by using disjunctive kriging as a non-linear estimator in comparison with traditional ordinary kriging by considering different cut-offs

Methodology

- ***Disjunctive kriging***
  - Disjunctive kriging allows predicting the regionalized variable and any (linear or nonlinear) function of it
  - There are many models for Disjunctive Kriging but here we considered bivariate Gaussian distributions
  - To apply such a bivariate Gaussian model, the random field Z has to be transformed into a normally-distributed random field by Gaussian anamorphosis or normal scores transformation
  - for each pair of locations **x** and **x+h** in R^d

$$(Y(\mathbf{x}), Y(\mathbf{x}+\mathbf{h})) \sim N(\boldsymbol{\mu}, \boldsymbol{\Sigma}(\mathbf{h})) \text{ with } \boldsymbol{\mu} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \text{ and } \boldsymbol{\Sigma}(\mathbf{h}) = \begin{pmatrix} 1 & \rho(\mathbf{h}) \\ \rho(\mathbf{h}) & 1 \end{pmatrix}$$

Methodology

- Disjunctive kriging (DK) is meant to predict a function of *Z*, which can also be written as a function of *Y*, say $\varphi(Y(\mathbf{x}))$
- Expanding j into Hermite polynomials $\{H_p : p \in \mathrm{N}\}$

$$\varphi(Y(\mathbf{x})) = \sum_{p=0}^{+\infty} \varphi_p H_p(Y(\mathbf{x}))$$

- by separately predicting each Hermite polynomial with simple kriging

$$[\varphi(Y(\mathbf{x}))]^{DK} = \sum_{p=0}^{+\infty} \varphi_p [H_p(Y(\mathbf{x}))]^{SK}$$

Methodology

- The rationale behind these expansions is that, in the bivariate Gaussian model, the Hermite polynomials form an orthonormal basis for the square-integrable functions of *Y*

$$\forall y \in \mathrm{R}, H_0(y) = 1$$

$$\forall p > 0, E\{H_p(Y(\mathbf{x}))\} = 0$$

$$\forall p, q > 0, \operatorname{cov}\{H_p(Y(\mathbf{x})), H_q(Y(\mathbf{x}+\mathbf{h}))\} = \begin{vmatrix} \rho(\mathbf{h})^p \text{ if } p = q \\ 0 \text{ otherwise} \end{vmatrix}$$

Methodology

- The simple kriging predictor of $H_p(Y)$ tends to its mean value (0) as p increases, so that the high-order terms in above equation become negligible and the expansion can be approximated by truncating at a finite order $P_{\max}$

$$[\varphi(Y(\mathbf{x}))]^{DK} \approx \sum_{p=0}^{P_{\max}} \varphi_p [H_p(Y(\mathbf{x}))]^{SK}$$

- Interestingly, as for linear kriging, it is possible to alleviate the assumption that the Hermite polynomials have a known (zero) mean value and to design an “ordinary disjunctive kriging” (ODK) predictor that assumes the mean values of these polynomials are unknown

Methodology

- Substitute ordinary kriging for simple kriging in the definition of the disjunctive kriging predictor

$$[\varphi(Y(\mathbf{x}))]^{ODK} = \sum_{p=0}^{+\infty} \varphi_p [H_p(Y(\mathbf{x}))]^{OK} = \varphi_0 + \sum_{p=1}^{+\infty} \varphi_p [\sum_{\alpha=1}^{n} \lambda_\alpha^{(p)}(\mathbf{x}) H_p(Y(\mathbf{x}_\alpha))]$$

$$\begin{aligned}
[\varphi(Y(\mathbf{x}))]^{ODK} &\approx \varphi_0 + \sum_{p=1}^{P_{\max}} \varphi_p [\sum_{\alpha=1}^{n} \lambda_\alpha^{(p)}(\mathbf{x}) H_p(Y(\mathbf{x}_\alpha))] + \sum_{p=P_{\max}+1}^{+\infty} \varphi_p \left[\sum_{\alpha=1}^{n} \frac{1}{n} H_p(Y(\mathbf{x}_\alpha))\right] \\
&\approx \sum_{p=1}^{P_{\max}} \varphi_p [\sum_{\alpha=1}^{n} (\lambda_\alpha^{(p)}(\mathbf{x}) - \tfrac{1}{n}) H_p(Y(\mathbf{x}_\alpha))] + \sum_{p=0}^{+\infty} \varphi_p \left[\sum_{\alpha=1}^{n} \frac{1}{n} H_p(Y(\mathbf{x}_\alpha))\right] \\
&\approx \sum_{p=1}^{P_{\max}} \varphi_p [\sum_{\alpha=1}^{n} (\lambda_\alpha^{(p)}(\mathbf{x}) - \tfrac{1}{n}) H_p(Y(\mathbf{x}_\alpha))] + \frac{1}{n} \sum_{\alpha=1}^{n} \varphi(Y(\mathbf{x}_\alpha))
\end{aligned}$$

Methodology

- The steps for ordinary disjunctive kriging are therefore the following ones:

❖ Transform the original data (associated with the random field *Z*) into normally-distributed data or normal scores (associated with the random field *Y*) and determine an experimental anamorphosis function [5, 12, 27].

❖ Model the anamorphosis function (hereafter denoted as f), which can be done by fitting a polynomial function

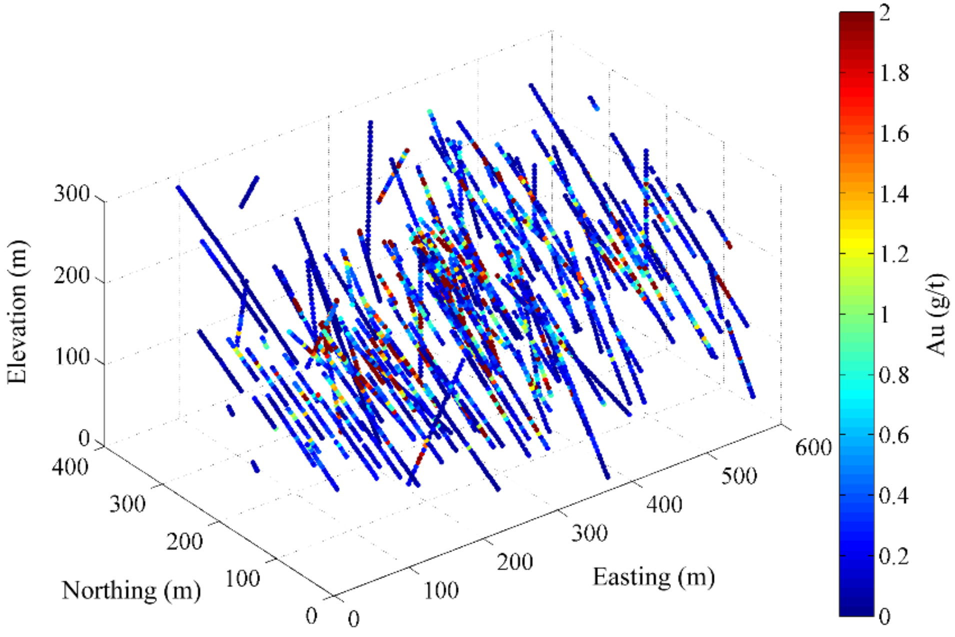
❖ Expand the function of *Y* of interest into Hermite polynomials, so as to determine the coefficients $\{j_p: p = 1 \ldots P_{max}\}$ up to some high order P_{max}.

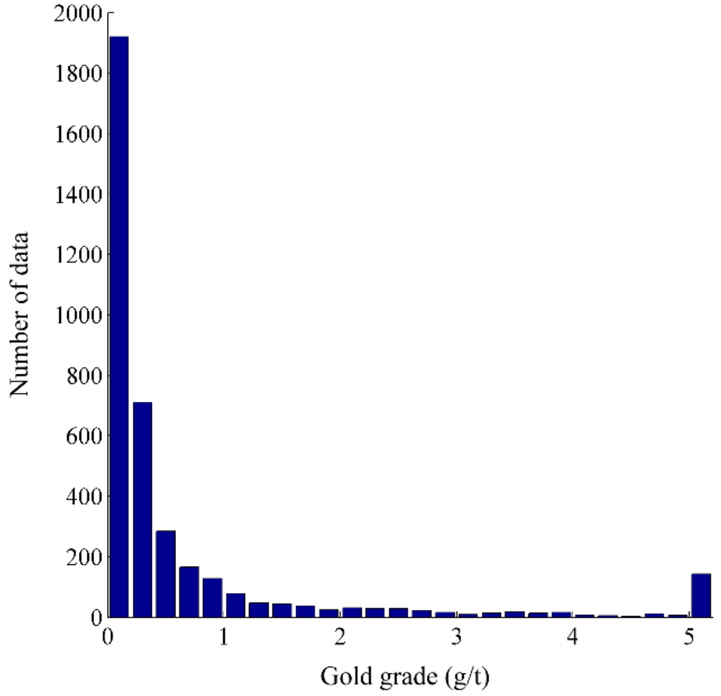
Methodology

- Model the spatial correlation structure of random field *Y*. This is classically done by calculating an experimental auto-covariance of the *Y*-data and fitting an auto-covariance model r(**h**); at this stage, the variogram can be used as an alternative tool to the auto-covariance, to ease the identification of the spatial correlation structure.
- For each target location **x**:
  - Search for neighboring data locations $\{\mathbf{x}_a: a = 1\ldots n\}$.
  - For $p = 1\ldots P_{max}$, perform ordinary kriging of $H_p(Y(\mathbf{x}))$ (the mean of $H_p(Y)$ is assumed unknown and its auto-covariance function is nothing else than $r(\mathbf{h})^p$
  - Calculate the ordinary disjunctive kriging predictor,

Case study (porphyry gold deposit)

- A disseminated porphyry gold deposit, the name, and location of which are undisclosed for confidentiality reasons. The data set consists of 3810 exploration drill hole samples composited at a length of 5 m, located in a parallelepiped region of size 600 m × 400 m × 300 m



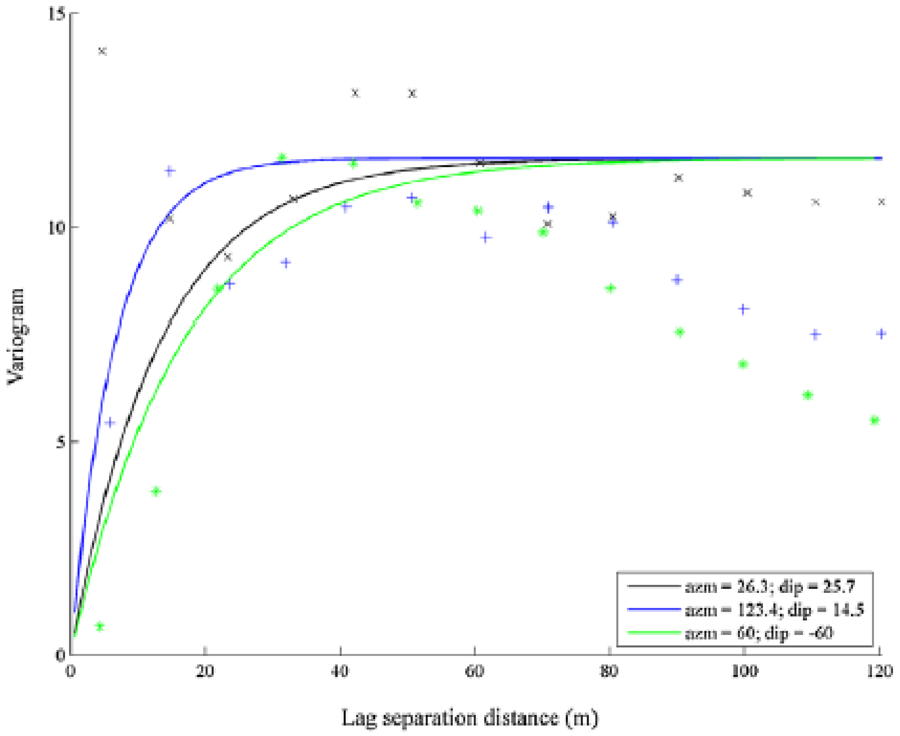


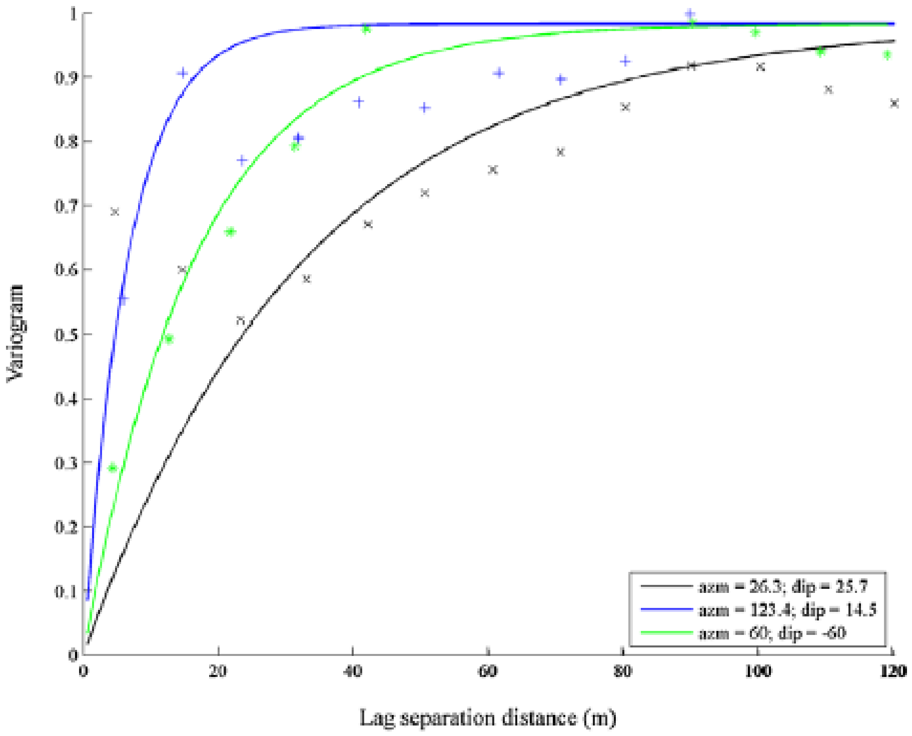
Case study

Number of data	3810
Minimum	0.010
Lower quartile	0.074
Median	0.198
Upper quartile	0.542
Maximum	86.62
Mean	0.956
Variance	11.52
Coefficient of variation	3.550

Case study

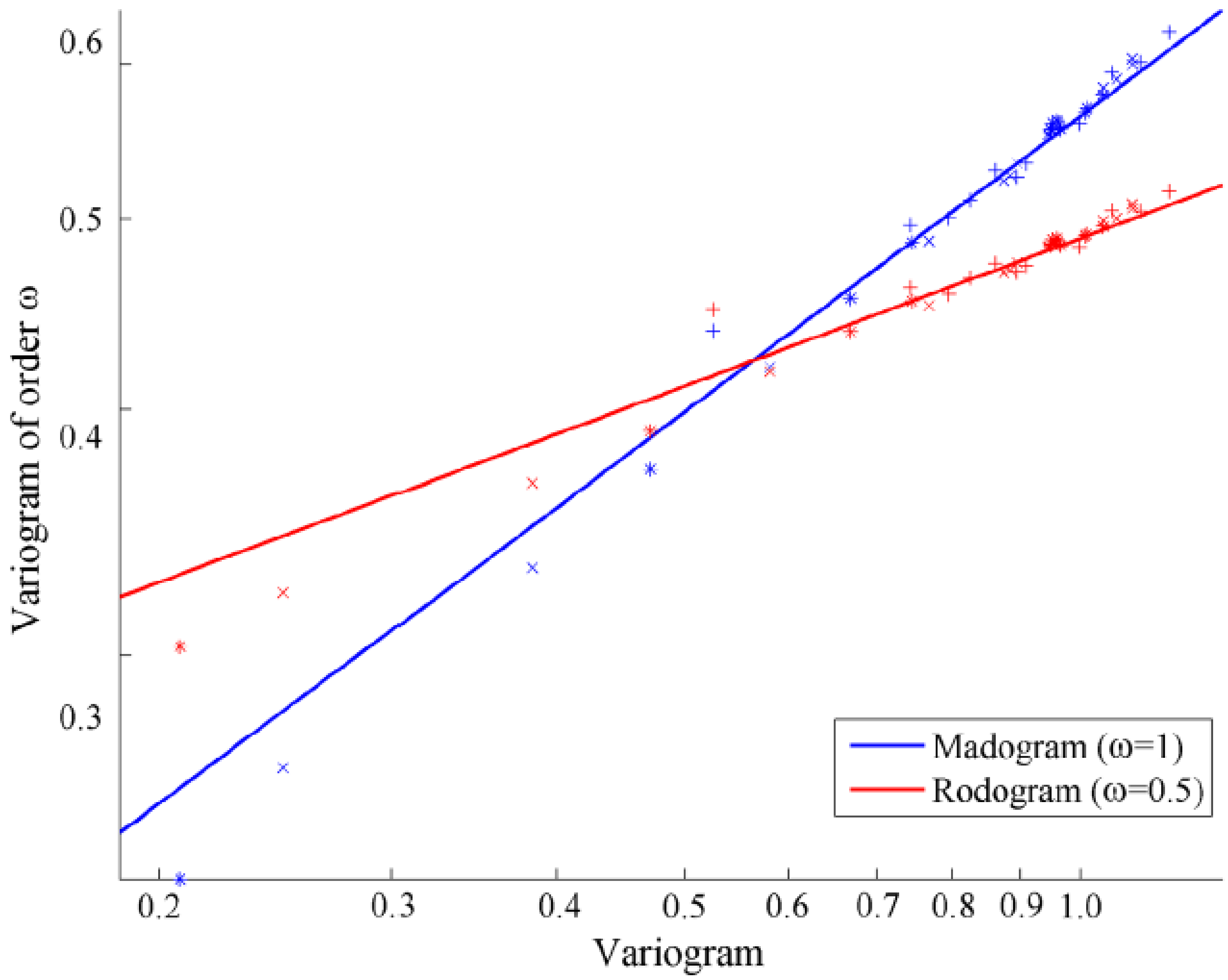
- The experimental variograms of the raw and normal scores data are calculated along three orthogonal directions, recognized as the main anisotropy directions and defined by rotating the coordinate axes by an azimuth of 60°, a dip of -60° and a plunge of 30°.





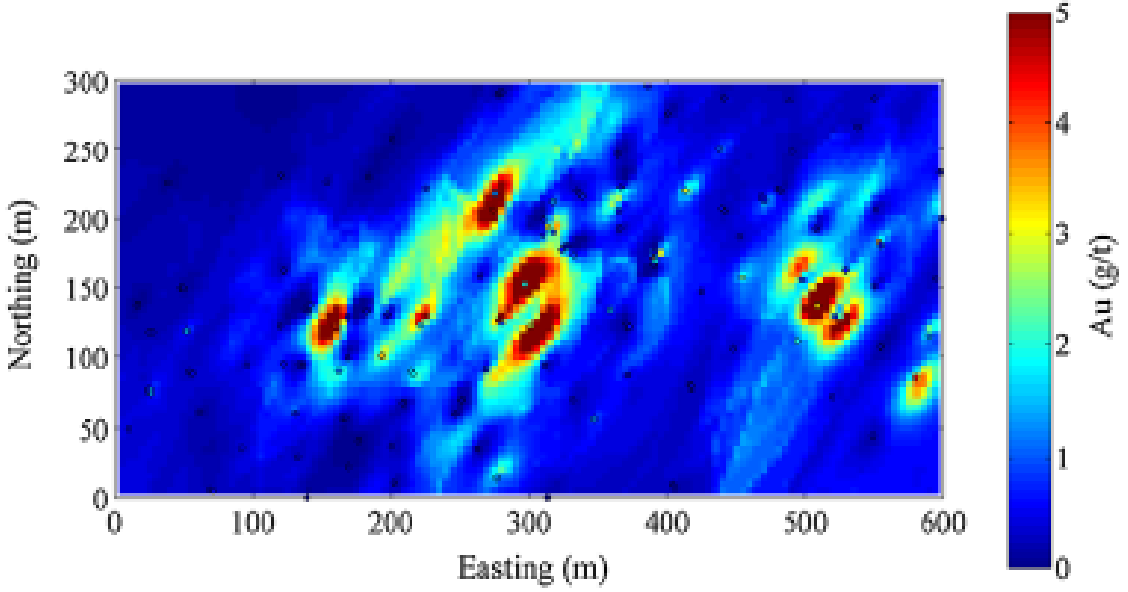
Case study

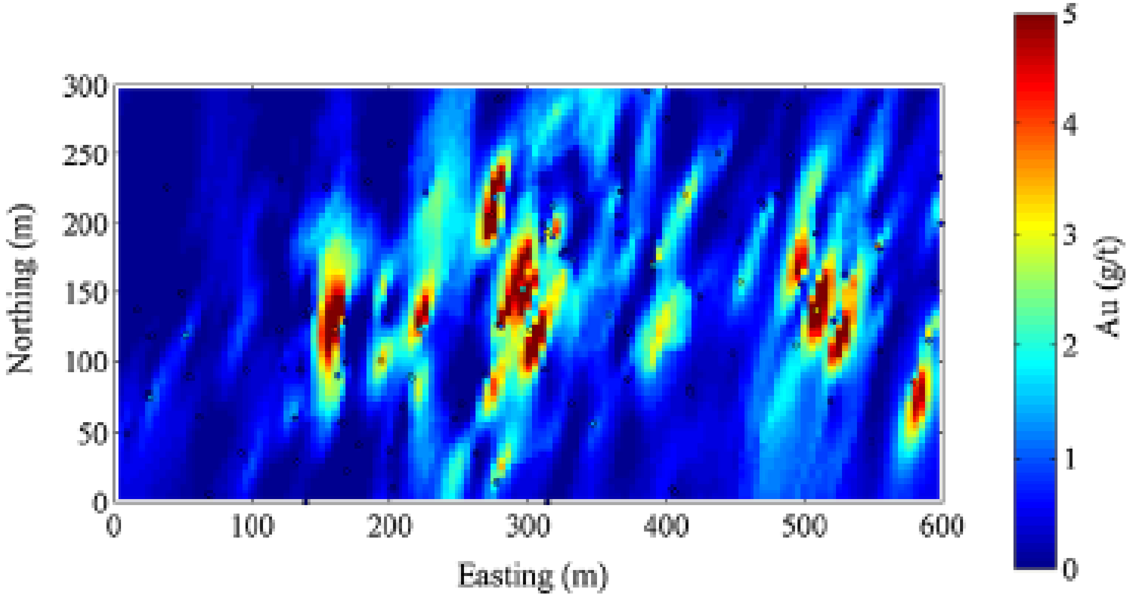
- The relationships between the variogram, madogram and rodogram that should be observed in the bivariate Gaussian model reasonably hold, except for the first two lags corresponding to distances less than 10 m



Case study

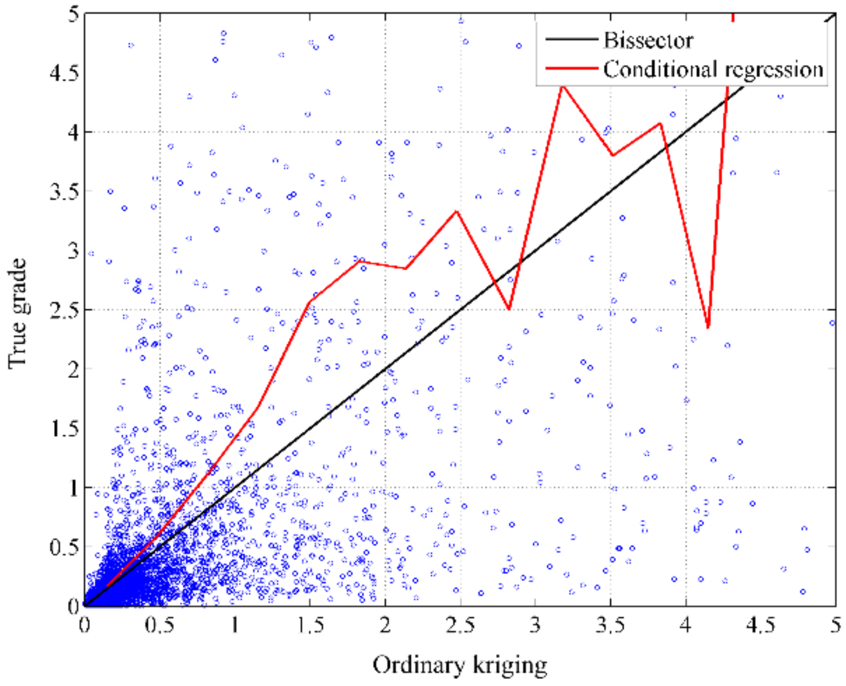
- ***Prediction of gold grade***
  - Ordinary kriging and ordinary disjunctive kriging are applied to predict the gold grade on a regular grid with mesh size 5 m × 5 m × 5 m, using the 50 drill hole data closest to the target grid node.

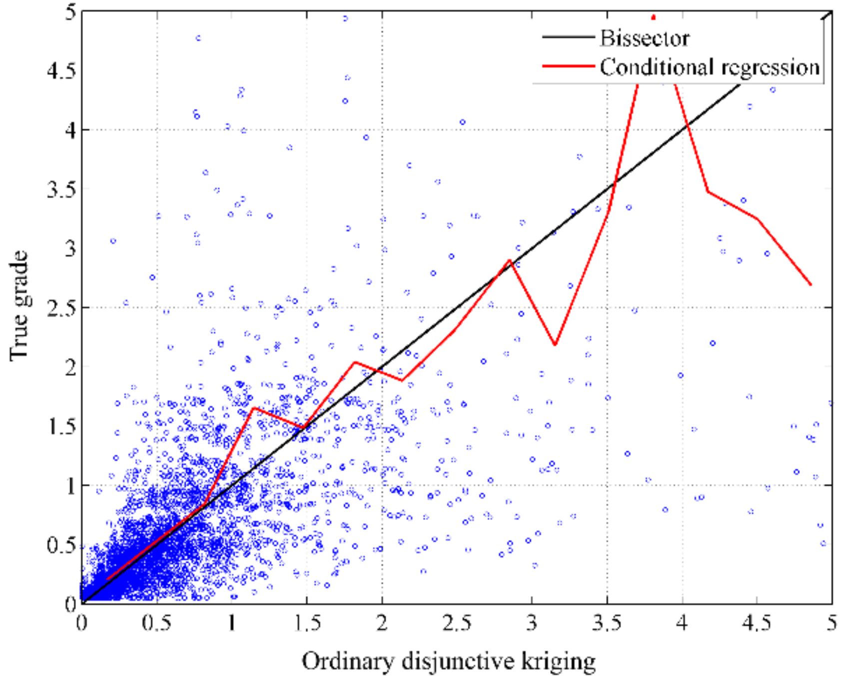




Case study

- ***Leave-one-out cross-validation***
    - The accuracy of the predictions is assessed through leave-one-out cross-validation, which consists in predicting the gold grade at each data location using the information of the remaining data





Case study

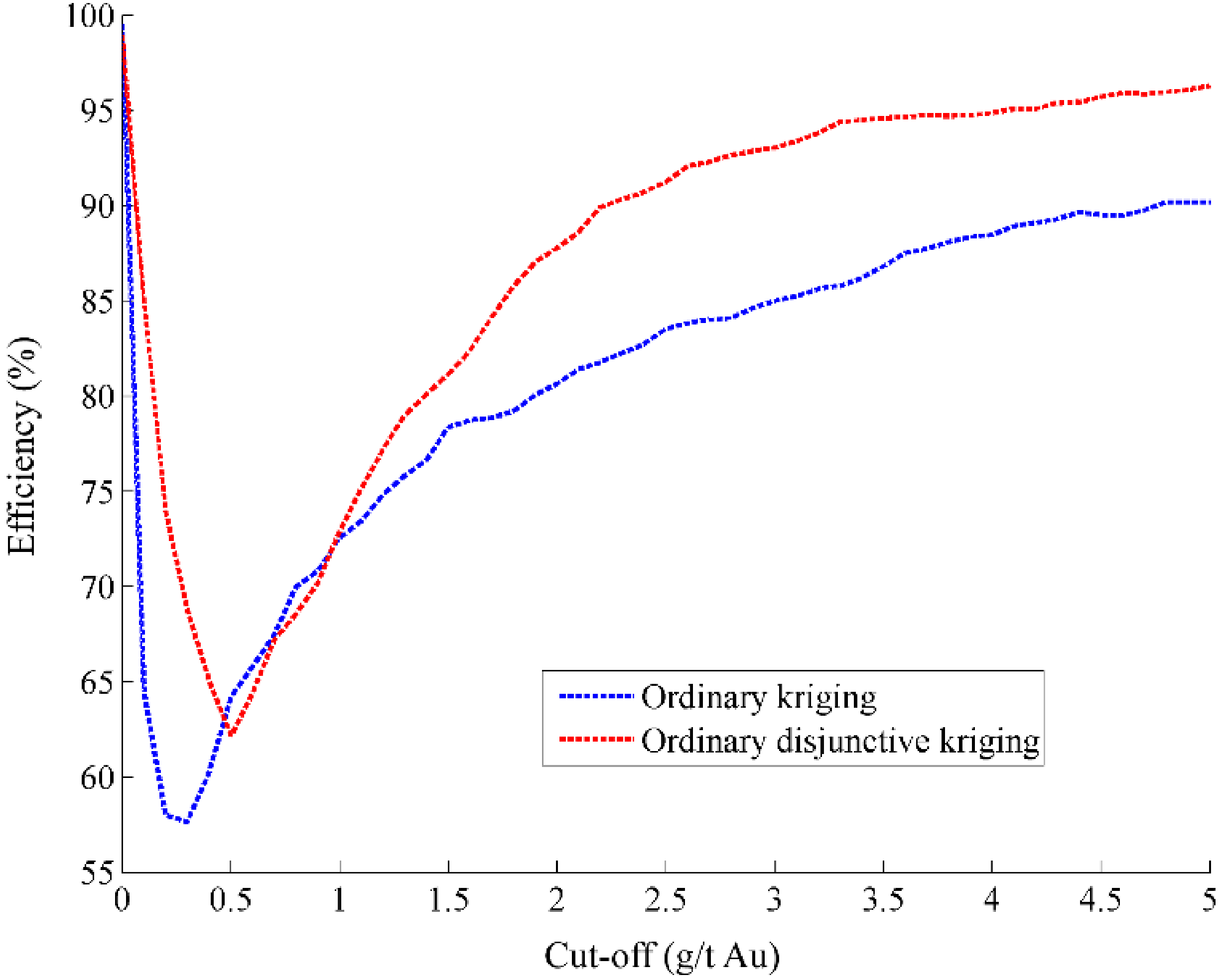
- The efficiency of classification can be measured through the following index:

$$Eff = 100 \times \frac{\text{number of correct classifications} - \text{number of misclassifications}}{\text{number of correct classifications} + \text{number of misclassifications}}$$

- This index is equal to 100 when all the blast holes are correctly classified and to 0 when the number of correct classifications equals the number of misclassifications, which would be the efficiency of a classifier that distinguishes between ore or waste by tossing a coin.

Case study

- Efficiency of classification



Conclusions

- This work compared the performance of traditional ordinary kriging with that of a non-linear predictor (ordinary disjunctive kriging) through a case study in mineral resources evaluation
- In the case study, the variable of interest (gold grade) is heavy-tailed and exhibits a few extreme values. In such a case, ordinary disjunctive kriging turns out to be significantly more accurate than ordinary kriging, yielding a better assessment of the mineral resources and a more efficient classification into ore and waste.
- Because of its greater complexity in comparison with linear kriging, disjunctive kriging has an incipient use in mining applications.
- A version based on ordinary kriging has been presented, which allows the mean value of the transformed data to vary in space, although this mean value remains constant locally (at the scale of the kriging neighborhood). Just like ordinary kriging, such an "ordinary disjunctive kriging" is well suited to the prediction of locally stationary random fields; furthermore, it is robust against moderate deviations from the assumed bivariate Gaussian distribution model.
- The prediction of grade can be upscaled directly, by averaging over a block support, as the grade is generally an additive variable.

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