On the Notion of Conditional Variance of Recoverable Ore Grade

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IAMG 2017

Introduction

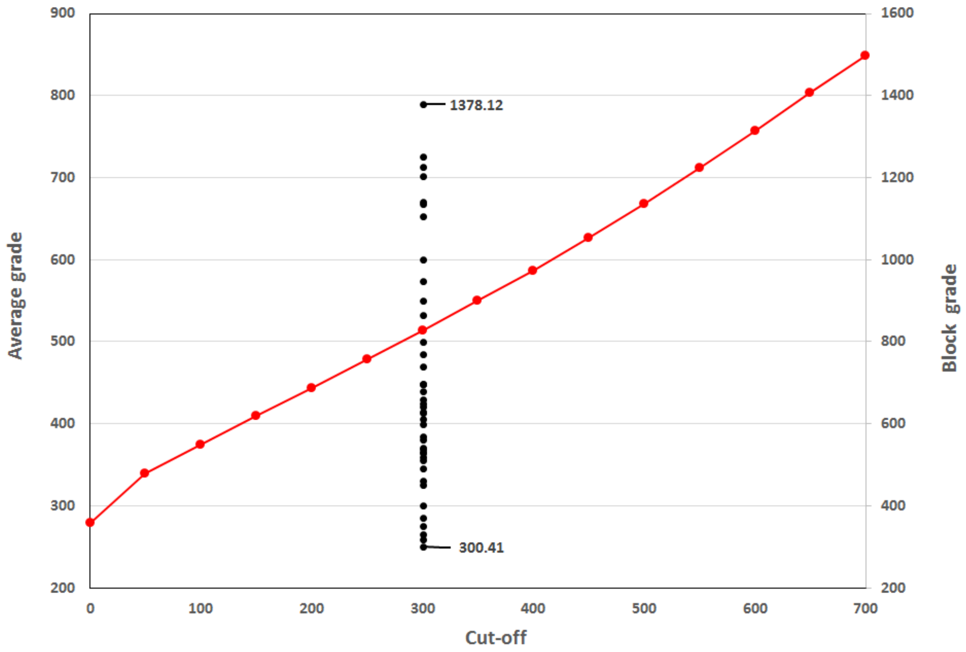
- The discrete Gaussian method (DGM) is a change of support approach that allows the assessment of the average tonnage T, metal Q and grade M above cut-off for a given support and cut-off grade z
- However, information about the variability around these average figures is not directly accessible
- Conditional simulations can be used but are time consuming and require more effort

Introduction

- The interest is on the dispersion of the recoverable grade Z around the average grade above cut-off M for a given SMU size v and cut-off grade z

$$M(z) = E[Z(v)/Z(v) \geq z]$$

$$\sigma_M^2(z) = E\left[\left(Z(v) - M(z)\right)^2 / Z(v) \geq z\right]$$



The Discrete Gaussian Method

- Assumes that the known drill hole data Z is a non-linear function of a discrete number of Gaussian values Y

$$Z(x) = \phi\big(Y(x)\big) = \sum_{n\geq 0} \phi_n H_n\big(Y(x)\big)$$

- Block grades are expressed as

$$Z(v) = \phi_v(Y_v) = \sum_{n\geq 0} \phi_n r^n H_n(Y_v)$$

- The relation between the drill hole data and the block grades is obtained through the relation between the Gaussian attributes Y and Y_v

The Discrete Gaussian

- The average grade above cut-off can be computed explicitly

$$M(z) = \phi_0 - \frac{1}{P(Y_v \geq y)} \sum_{n \geq 1} \phi_n r^n \frac{H_{n-1}(y) g(y)}{\sqrt{n}} \qquad z = \phi_v(y)$$

- Aim is to obtain an explicit analytical expression for the dispersion

$$\sigma_M^2(z) = E\left[\left(Z(v) - M(z)\right)^2 / Z(v) \geq z\right]$$

$$= E[Z(v)^2 / Z(v) \geq z] - M(z)^2$$

The Discrete Gaussian

- Using the development into Hermite polynomials

$$\sigma_M^2(z) = \frac{1}{P(Y_v \geq y)} \sum_{n,m\geq 0} \phi_n \phi_m r^n r^m \; U_{n,m}(y) - \; M(z)^2 \qquad z = \phi_v(y)$$

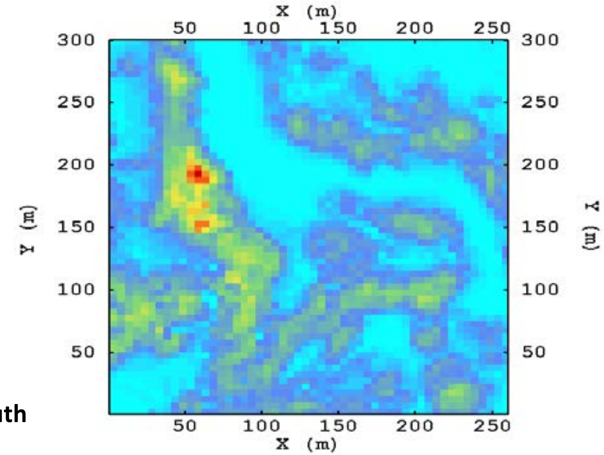
$$U_{n,m}(y) = \int_y^{+\infty} H_n(u) H_m(u) g(u) du$$

- At zero cut-off the conditional variance is equal to the SMU variance $Var\big(Z(v)\big)$

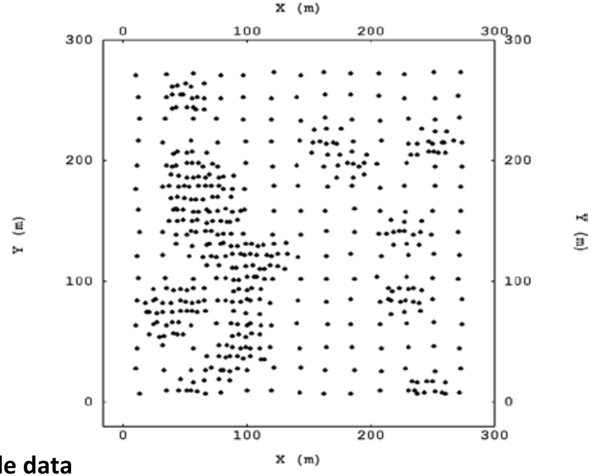
A case study

Walker Lake data set

- Exhaustive SMU grades are known
- SMU size is 5 m by 5 m
- Drill hole data exhibits a nominal drill spacing of 20 m by 20 m with some infill drilling



Ground truth



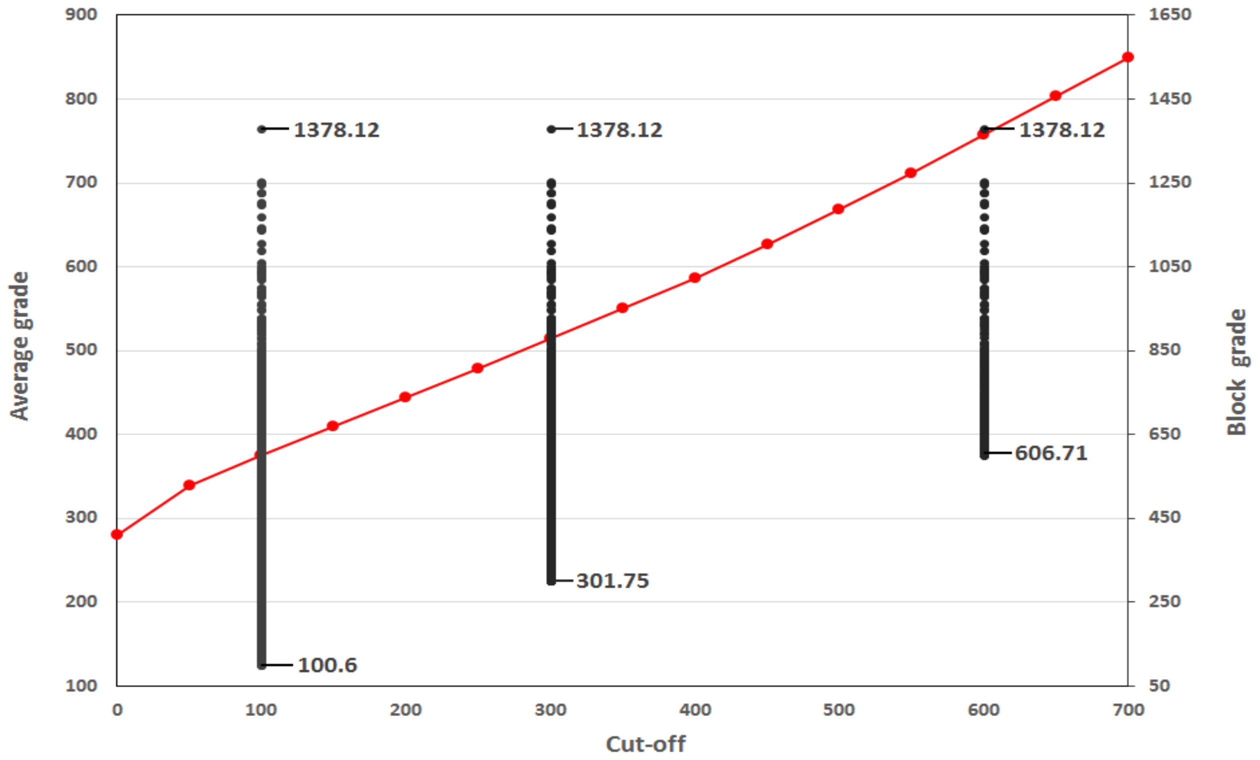
Drill hole data

Walker Lake data set

CUT-OFF		DISPERSION	RELATIVE ERROR (%)
250	Ground truth	34009.31	1.77
	Estimated	33419.09	
300	Ground truth	31132.76	1.49
	Estimated	30676.30	
350	Ground truth	28513.38	0.43
	Estimated	28392.54	

The conditional variance is a decreasing function of the cut-off grade

Walker Lake data set



Control Charts for Block Grades

Control Charts for Block Grades

- Upper and lower control limits can be obtained as in traditional control charts

$$[M(z) - k\sigma_M(z), M(z) + k\sigma_M(z)] \qquad k > 0$$

- Probability of exceedance

$$P(|Z(v) - M(z)| \geq k\sigma_M(z) / Z(v) \geq z)$$

can be computed regardless of the the distribution of $Z(v) / Z(v) \geq z$

Control Charts for Block Grades

- For instance, for $z \in [M(z) - k\sigma_M(z), M(z) + k\sigma_M(z)]$

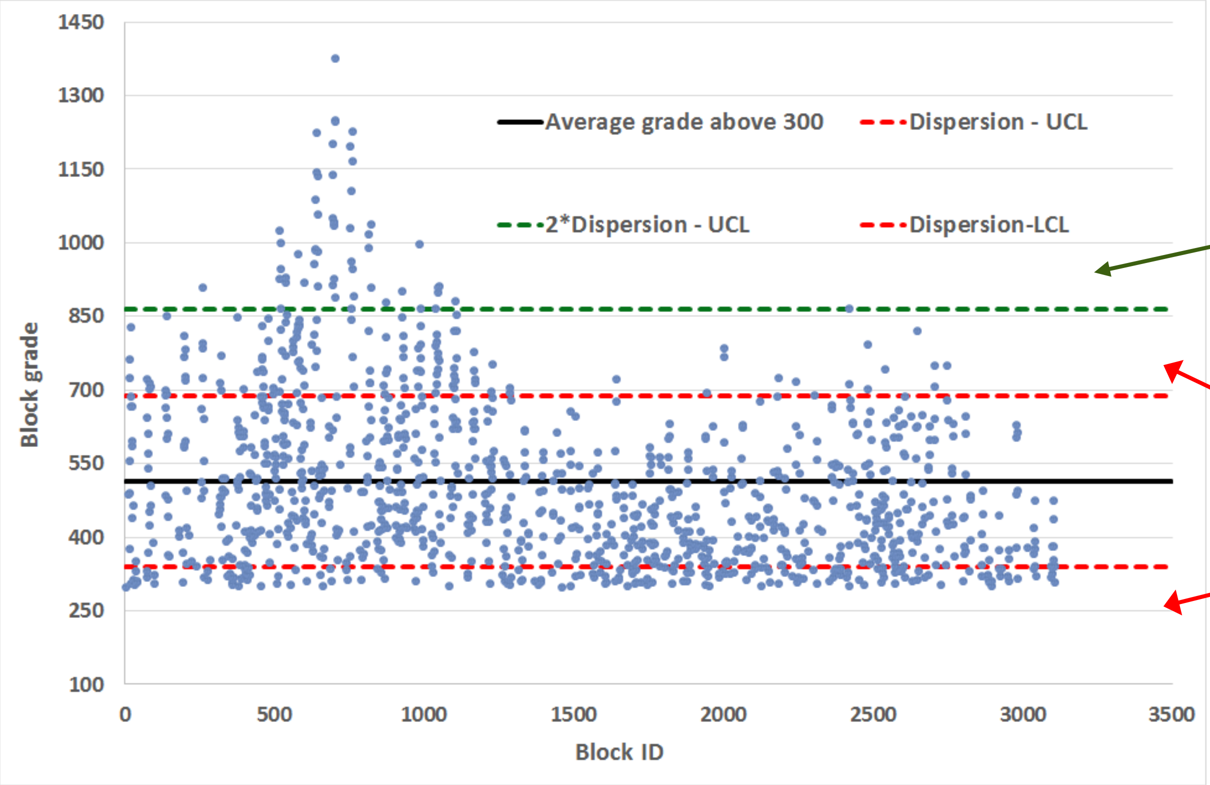
$$P(|Z(v) - M(z)| \geq k\sigma_M(z)/Z(v) \geq z) = 1 - \frac{1}{\left(1 - F(y)\right)}[F(b) - F(a)]$$

$$b = \phi_v^{-1}\left(M(z) + k\sigma_M(z)\right)$$

$$a = \phi_v^{-1}(z)$$

$$z = \phi_v(y)$$

Control Charts for Block Grades



Ground truth: 5%
Estimated: 3%

Ground truth: 29%
Estimated: 26%

Selection on Estimates

Selection on Estimates

- In practice is not possible to select on block values $Z(v) \geq z$ and an estimate $Z^*(v)$ is used instead.

- Interest is on the effective grade and its corresponding dispersion

$$M^*(z) = E[Z(v)/Z^*(v) \geq z] \qquad z = \phi_v^*(y)$$

$$\sigma_{M^*}^2(z) = E\left[\left(Z(v) - M^*(z)\right)^2 / Z^*(v) \geq z\right]$$

- Modelling of the information effect allows to derive the conditional variance

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